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Finiteness of a set of non-collinear vectors generated by a family of linear operators

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Abstract

Let $\mathbb{R}^n$ be a real n-dimensional space, let $\{A(x) \mid x \in X\}$ be a family of $m = |X|$ linear operators in $\mathbb{R}^n$, and let K_r be a sharp polyhedral cone formed by a set of $r$vectors, $K_r \subset \mathbb{R}^n$. Let K_r be invariant under $\{A(x) \mid x \in X\}$, i.e. $K_r A(x) = K_r$, for $x \in X$. We study a maximum set of non-collinear vectors derived from a vector $\mathbf{h} \in K_r$ by the family $\{A(x) \mid x \in X\}$ in this paper. It is shown that there is a function $f(n, m, r)$ such that this set of non-collinear vectors is finite iff the cardinality of this set is not greater than $f(n, m, r)$. This result can be used for solving the following problem: when does a channel simulated by a probabilistic automaton have a finite set of states? © 1999 Elsevier Science Inc. All rights reserved.

Keywords: Linear operator; Eigenvectors; Polyhedral cone; Probabilistic automaton

1. Introduction

A channel, simulated by a finite probabilistic automaton [1], can be described by an initial vector and a family of matrices. In [1, p. 66, Theorem 2.4], an algorithm is given to decide whether the set of channel states is finite. But it

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